23. Don ate $x-27$ cherries and Juan ate $x-11$ cherries. Since $x-27 \ge 10$ and $x-11 \ge 10$, $x \ge 37$. In addition, $x-11+x-27 \le x-1$, so $x \le 37$. Therefore, $x = 37$.

A) 37 B) 38 C) 39 D) 49

23. A

24. If we subtract a and b from 200, we subtract the pets with scales **and** gills twice. Adding them back once, we have $200-a-b+c$ with neither.

A) $200-a-b$ B) $200-c$ C) $200-a-b-c$ D) $200-a-b+c$

24. D

25. Since $xy = 144$ and $y = 3x-6$, $3x^2-6x = 144$. Hence, $x^2-2x-48 = 0$. Thus, $(x+6)(x-8) = 0$ and $x = -6$ or 8. Since $y < x$, $x = -6$.

A) 18 B) 8 C) –6 D) –24

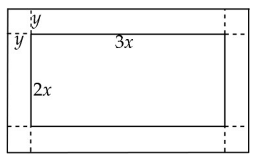
25. C

26. If $x+y = 2$, $y = 2-x$ and $20x+50y = 20x+50(2-x) = 100-30x < 100$. Similarly, $x = 2-y$ and $20x+50y = 40+30y > 40$.

A) 35 B) 65 C) 105 D) 140

26. B

27. If the border's width is y, the area of the border is $2y(2x)+2y(3x)+4y^2 = 4y^2+10xy$. We are given the area is $14x^2$, so $4y^2+10xy = 14x^2$. Thus, $(2y+7x)(y-x) = 0$. Since $y > 0$, $y = x$.



A) $0.5x$ m B) x m C) $1.5x$ m D) $2x$ m

27. B

28. 10^{2019} has 2020 digits. Subtracting 2019 from 10^{2019}, the result is 999 999 999 . . . 999 997 981. That's 2015(9)+7+9+8+1 =18160.

A) 2019 B) 18 160 C) 18 161 D) 18 169

28. B

29. The percent of sugar is $(10x+20y)/(x+y)$. Set this equal to z and solve: $10x+20y = xz+yz$, so $(10-z)x = (z-20)y$ and $x/y = (z-20):(10-z)$.

A) $(20-z):(z-10)$ B) $(10-z):(z+20)$
C) $(z+10):(20-z)$ D) $(z+20):(10-z)$

29. A

30. If x, y, and z are prime, the whole-number divisors of the product xyz are 1, x, y, z, xy, xz, yz, and xyz. The product of these is $x^4y^4z^4$.

A) xyz B) $x^2y^2z^2$ C) $x^3y^3z^3$ D) $x^4y^4z^4$

30. D

The end of the contest **A**

Visit our Web site at http://www.mathleague.com

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Math League Press, P.O. Box 17, Tenafly, New Jersey 07670-0017

Information & Solutions

A

Spring, 2019

Directions for Grading

- **Date** You may give this contest any time after April 15. The *Algebra Course 1 Contest* is for use in your own school or district. We've enclosed a registration form for next year. Instructions for optionally submitting results are included on a separate sheet entitled "*Using the Score Report Center*."
- **Urgent questions?** Write to comments@mathleague.com, or call 1-201-568-6328 or 1-516-365-5656.
- **Scores** Remind students that *this is a contest, and not a test*—there is no "passing" or "failing" score. Few students score as high as 24 points (80% correct); students with half that, 12 points, *should be commended!*
- **Solutions** Detailed solutions appear in each question box, and letter answers are in the *Answers* columns on the right. You may copy this solution key and give a copy to every student who took this contest.
- **Awards** The original contest package contained 1 book award (and a bookplate you should affix to the book's inside front cover) for the 1st place student. We also enclosed 5 *Certificates of Merit*—1 each for the runner-up on each grade level, plus extras for ties.
- **Additional Book Awards & Additional Certificates** If you want to give more than 1 book award, you may purchase additional books as described below. Do you need more Certificates of Merit? If so, send your name, school, and school mailing address to our mailer at: ***Math Certificates, P.O. Box 17, Tenafly, NJ 07670-0017.*** *Include a self-addressed, stamped envelope (**2 stamps required**) large enough to hold certificates.*

The school's top scorer will receive the book *Math Contests—High School (Vol. 4)*. Other high scorers will receive Certificates of Merit. In any one school year, no student may win both a book and a certificate. The book and certificates were in the original contest package.

If needed, duplicate book awards may be ordered as described below.

1. If $a = 2$, $r = 0$, $t = 1$, and $s = 9$, then $s + t + a + r + t = 9 + 1 + 2 + 0 + 1 = 13$.
A) 0 B) 12 C) 13 D) 21
1. C

2. There were a ants in my ant farm. They have $6a$ legs. After 3 ants leave, the remaining ants have $6a - 18 = 6(a - 3)$ legs.
A) $6a - 3$ B) $6(a - 3)$ C) $6a - 3a$ D) $a^6 - 3$
2. B

3. Regroup: $(6x^2 + 2x^2 + 4x^2) + (4x + 2x + 6x) - (5 + 3 + 1 + 3 + 5)$.
A) $36x - 17$ B) $24x - 9$
C) $12x^2 + 12x - 12$ D) $12x^2 + 12x - 17$
3. D

4. $(x - y)(x + y) = x^2 + xy - xy - y^2 = x^2 - y^2$.
A) $x^2 - y^2$ B) $x^2 - 2xy + y^2$ C) $x^2 + 2xy + y^2$ D) $x^2 + y^2$
4. A

5. $(x - y)(x + y)(x - y) = (x^2 - y^2)(x - y) = x^3 - x^2y - xy^2 + y^3$.
A) $x^3 - y^3$ B) $x^3 - x^2y - xy^2 + y^3$
C) $x^3 + y^3$ D) $x^3 + x^2y + xy^2 + y^3$
5. B

6. Since $-s^2 \le 0$ for all real values of s, $-s^2 - 1 < 0$ for all real values of s.
A) $-s^3 - 1$ B) $(-s)^3 - 1$ C) $-s^2 - 1$ D) $(-s)^2 - 1$
6. C

7. The integer solutions of $(x^2 - 1)(x^2 - 2)(x^2 - 3)(x^2 - 4) = 0$ are ± 1, ± 2.
A) 2 B) 4 C) 6 D) 8
7. B

8. If x, y, and z are distinct prime numbers, the least common multiple of $x^2y^3z^4$ and $x^4y^3z^2$ must contain the highest power of each prime.
A) $x^8y^9z^8$ B) $x^6y^6z^6$ C) $x^4y^3z^4$ D) $x^2y^3z^2$
8. C

9. $((x^3 + x^3) \times x^3)^3 = (2x^3 \times x^3)^3 = (2x^6)^3 = 2^3x^{18} = 8x^{18}$.
A) $2x^{18}$ B) $8x^{18}$ C) $8x^{27}$ D) x^{54}
9. B

10. In my jar, there are $3b$ red beans, 5b green beans, $6b$ orange beans, for a total of $14b$ beans. If $b = 3$, the total number of beans would be 42.
A) 35 B) 42 C) 60 D) 90
10. B

11. $2x - 2.5 = \pm 4$, so $x = 3.25$ or -0.75. The sum of the solutions is 2.5.
A) 2 B) 2.5 C) 3.75 D) 4
11. B

12. The roots of $(x - 7)(x + 4) = 0$ are 7 and -4. Their difference is 11.
A) 3 B) 4 C) 7 D) 11
12. D

2018-2019 ALGEBRA COURSE 1 CONTEST SOLUTIONS — Answers

13. Today Li turned 42 and Mae turned 8. In x years, we want $42 + x = 3(8 + x)$. Solving, $x = 9$. Therefore, Mae will be 17.
A) 9 B) 17 C) 26 D) 51
13. B

14. Three crates contain $3b$ boxes, and three boxes contain $3bp$ packages. If each package holds 4 bulbs, three crates contain $12bp$ bulbs.
A) $12bp$ B) $\frac{3bp}{4}$ C) $\frac{4bp}{3}$ D) $\frac{bp}{12}$
14. A

15. Before the wave hit, $a = 3b$. After the wave hit, $(a - 3)/(b + 1) = 5/2$. Combining these equations, $(3b - 3)/(b + 1) = 5/2$. Simplifying, $6b - 6 = 5b + 5$. Solving, $b = 11$. Since $a = 3b$, $a = 33$. Thus, Avi had built 33 sand castles before the wave hit.
A) 11 B) 12 C) 30 D) 33
15. D

16. If $135 \times (46 + 2) = (135 \times 46) + 270 = a + 270$.
A) $a + 2$ B) $a + 92$ C) $a + 94$ D) $a + 270$
16. D

17. If $3x + 8y = 21$ and $8x + 3y = 23$, $11x + 11y = 44$ and $x + y = 4$.
A) 2 B) 4 C) 11 D) 22
17. B

18. If the hands on a circular clock start at midnight, 1000 hours later is 83 full times around and then one-third more, which is 4 hours.
A) 2 B) 4 C) 8 D) 12
18. B

19. If $x = 3$, the value of $|20 - 7x|$ is 1.
A) 1 B) 2 C) 3 D) 6
19. A

20. If Sy can shovel snow from half of a driveway in 2 hours, and Ty can shovel snow from one quarter of the driveway in 2 hours, together they shovel three-quarters of the driveway in 120 minutes or one quarter in 40 minutes or four-quarters in 160 minutes.
A) 120 B) 160 C) 180 D) 360
20. B

21. Of the bottles that Viola collects, 80% are green. Of the green bottles, 30% held perfume 45% held spices. Thus, 25% of the green bottles held pills. Since 25% of 80% is 20%, and 20% of her bottles is 25, 100% of her bottles is 125.
A) 75 B) 100 C) 120 D) 125
21. D

22. Clearing fractions, $2x^2 - y + 3x^2 = 4$; $y = 5x^2 - 4$.
A) $4 - x^2$ B) $4 + x^2$ C) $5x^2 - 4$ D) $4 - 5x^2$
22. C