Math League News

■ Our Calculator Rule Our contests allow both the TI-89 and HP-48. You may use any calculator without a QWERTY keyboard.

■ Send Your Comments to comments@mathleague.com. View results at www.themathleague.com before they arrive in the mail.

■ Upcoming Contest Dates & Rescheduling Contests Future HS contest dates (and alternate dates), all Tuesdays, are Jan 10 (3), Feb 14 (7), & Mar 13 (6). (Each alternate date is the preceding Tuesday.) If vacations, school closings, or special testing days interfere, please reschedule the contest. Attach a brief explanation, or scores may be considered unofficial. We sponsor an *Algebra Course I Contest* in April, and contests for grades 4, 5, 6, 7, and 8. Get information and sample contests at *www.themathleague.com*.

■ Contest Dates for 2012-2013 and change in alternate date policy: HS contest dates for the next school year (and alternate dates), all Tuesdays, are October 16, 2012 (October 23), November 13, 2012 (November 20), December 11, 2012 (December 18), January 8, 2013 (January 15), February 12, 2013 (February 19), and March 12, 2013 (March 19). Please note that starting in October 2012, each alternate date will be on the Tuesday **following** the official date!

■ T-Shirts Anyone? We're often asked, "are T-shirts available? The logo lets us recognize fellow competitors!" Good news — we have MATH T-shirts in a variety of sizes at a **very** low price. Use them as prizes for high or even perfect scores, or just to foster a sense of team spirit! The shirts are of grey material and feature a small, dark blue logo in the "alligator region." A photo of the shirt is available at our website. There's one low shipping charge per order, regardless of order size. To order, use our website, www.themathleague.com.

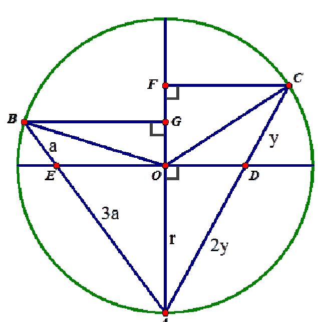
■ New: Contests for iPads and iPhones! We are pleased to announce that we now have iPad/iPhone versions of ALL of our prior contests for grades 4, 5, 6, 7, and 8 and the Algebra contests available now, including last year's contests. We are not sure when high school contests will be available, but we are working on it! The link to these iPad/iPhone applications is on the home page of our website, www.themathleague.com. Take note of our current special offer: access to **all** past contests at any selected grade level for **all** students at a given school for the low, low price of only $9.95 for the year!

■ General Comments About Contest #3: Jon Graetz said, "Great contest! Numbers 5 and 6 were especially difficult, but quite interesting." Dick Gibbs said, "I enjoyed contests 2 and 3. Contest 1 was an easy start, but I'm glad that 2 and 3 picked up the pace a bit." Mark Luce said, "A lovely contest." Fred Harwood said, "Good contest as shown by few leaving early. Have a great Christmas everyone. Thank you for the nice present."

■ Question 3-2: Appeal (Denied) Kevin Horstman appealed on behalf of a student who answered with "1, 2, 3, 5, 8, 8, 8." Since the appealed response includes 7 integers and the question specifies that the answer should include only 5 integers, the appeal is denied.

■ Question 3-3: Comment and Appeals (Accepted and Denied) Dick Gibbs said, "I thought that 3-3 was harder than a level 3 problem. It's tempting to guess that the product is maximized when the factors are equal, but the proof is tricky." Janet Kagan wondered whether a student who wrote 0.3 with a repetend bar should be given credit. Yes, that answer would be acceptable. April Parker appealed on behalf of a student who wrote ".33" in answer to this question. Approximate answers must be correct to at least 4 significant digits, and all digits written must be correct. Since 0.33 has only 2 significant digits, it is not a correct response. (Had the student written "0.3333," credit would be given for the response.)

■ Question 3-6: Comments, Appeals (Accepted and Denied) and Alternate Solutions Mark Luce said, "I particularly liked the tough geometry problem (Problem 6), even though only one of my students got it." Mary Elizabeth Harmon said, "Number 6 was really difficult and none of my students worked it correctly through to the end. They were on the right track, but obviously not right enough." Ed Groth appealed on behalf of a student whose answer was "$3y{:}4a$." Unless otherwise indicated, a numerical answer is always required on our contests. In addition, any answer that would trivialize the question or show that the student has made no attempt to actually solve it cannot be accepted. The student's answer is insufficient for both of these reasons, so the appeal is denied. Janet Kagan appealed on behalf of a student whose answer was "$3:2\sqrt{2}$," which is mathematically equivalent to the correct answer. We do not require that radical expressions be simplified or that fractions be rationalized, so the answer is correct. A number of our advisors wondered whether there were alternatives to our official solution; fortunately, we had no shortage of alternatives submitted! Jon Graetz submitted: Let the given circle be the unit circle centered on the origin. Call the outer endpoints of the chords A and B, and the common endpoint C. From A and B, drop perpendiculars to the y-axis. AC is the hypotenuse of a right triangle. The hypotenuse is divided by the x-axis 3/4 along its length. By similar triangles, the vertical leg is also divided 3/4 along its length. Since the lower portion is of unit length, the upper portion has length 1/3, and A is located at $(\sqrt{8}/3,\ 1/3)$. By the same plan, C is at $(\sqrt{3},1/2)$. Use distance formula to find AC = $2\sqrt{6}/3$ and BC = $\sqrt{3}$. Squaring each to compare lengths, $AC^2 = 24/9$ and $BC^2 = 3 = 24/8$, so AC < BC. BC/AC = $3\sqrt{2}/4$. Dick Gibbs submitted: Let r be the radius of the circle and let the 4 segments of the horizontal diameter have lengths (from left to right) of x, $r-x$, $r-z$, and z. By the intersecting chords (or power of a point) theorem, $3a^2 = x(2r-x)$ and $2y^2 = z(2r-z)$. By Pythagoras, $(r-x)^2 + r^2 = 9a^2$ and $(r-y)^2 + r^2 = y^2$. Simple manipulation gives $r^2 = 3y^2 = 6a^2$. So $y^2 = 2a^2$, $y/a = \sqrt{2}$, and $3y/4a = 3\sqrt{2}/4$. Ted Heavenrich and Jeff Schwartzman suggested similar solutions. Rhonda de la Mar submitted: The student that got the question right had a really nice solution using the cosine ratios. cos $s = r/3a$ and cos $s = 4a/2r$, so setting them equal yields $12a^2 = 2r^2$. Similarly on the other side cos $t = 3y/2r$ and cos $t = r/2y$. This yields $2r^2 = 6y^2$. These two equations give you that $12a^2 = 6y^2$ and therefore $y/a = \sqrt{2}$. You need the ratio of $3y/4a$ so the solution is $3\sqrt{2}/4$. Chip Rollinson submitted a similar solution that uses the same triangles, but using similar triangles instead of trig functions to find the relationship. He also submitted yet another alternate solution from one of his students:



$$\frac{AD}{DC}=\frac{AO}{OF}\Rightarrow\frac{2y}{y}=\frac{r}{OF}\Rightarrow OF=\frac{r}{2}.\quad FC=\sqrt{r^2-\left(\frac{r}{2}\right)^2}=\frac{r\sqrt{3}}{2}.$$

$$AC=\sqrt{FC^2+FA^2}=\sqrt{\left(\frac{r\sqrt{3}}{2}\right)^2+\left(\frac{3r}{2}\right)^2}=r\sqrt{3}.$$

Similarly on the left side,

$$AB=\frac{2\sqrt{6}}{3}r.$$

$$\therefore\frac{AC}{AB}=\frac{r\sqrt{3}}{\frac{2\sqrt{6}}{3}r}=\frac{3\sqrt{2}}{4}.$$

Statistics / Contest #3

Prob #, % Correct (*all reported scores*)

3-1	75%	3-4	41%
3-2	86%	3-5	34%
3-3	69%	3-6	4%