Math League News

■ Our Calculator Rule Our contests allow both the TI-89 and HP-48. You may use any calculator without a QWERTY keyboard.

■ Send Your Comments to comments@mathleague.com.

■ Contest Dates Future HS contest dates (and alternate dates), all Tuesdays, are December 13 (Dec. 20), January 10 (Jan. 17), February 7 (Feb. 14), and March 14 (Mar. 21). (Each alternate date is the Tuesday following the official date.) For vacations, special testing days, or other *known* disruptions of the normal school day, please *give the contest on the following Tuesday.* If your scores are late, please submit a brief explanation. We reserve the right to refuse late scores lacking an explanation. We sponsor an *Algebra Course I Contest* in April, as well as contests for grades 4, 5, 6, 7, & 8. See *www.mathleague.com* for information.

■ Regional Groupings Within guidelines, we try, when possible, to honor regional grouping requests for the next school year.

■ What Do We Print in the Newsletter? Space permitting, we print every solution and comment we receive. We prepare the newsletter early, so we can use only what we have at that time.

■ How Do I Change the Spelling of a Student Name? Please note that an advisor can always return to the Score Report Center to change the spelling of a student's name or to correct a score. We stay out of the loop on such changes. Any advisor noticing a need for such changes should feel free to make them directly.

■ Can I Add Additional Names and Scores to an Earlier Contest? One advisor asks, "Since some students did very well in the second contest, can we add their names (with the scores) to the Contest 1 report?" We always allow adding additional names and scores to an earlier contest as long as the additions do not affect the team total previously submitted for the earlier contest.

■ Administer This Year's Contests Online Any school that is registered for any of our contests for the 2016-2017 school year may now register at **www.online.mathleague.com** for the 2016-2017 Online Contests at no cost. The advantages of administering the online versions of our contests rather than the paper and pencil ones are that you do not have to grade your students' papers and that you do not have to submit any scores at our Score Report Center ~ these tasks are done automatically for you when your students take our contests online. If you decide to use this free service, you must set up your account and set the day you will administer each contest at least one day in advance of the actual contest date.

■ General Comments About the Contest James Conlee said, "Another great contest. This contest was very accessible to students of all levels." David Pomerance said, "I enjoyed the six questions this month!" John Cirillo said, "Perhaps the most difficult contest I've witnessed in my 37 years as advisor. Questions 4-6 were time consuming for my students. No one finished the test in the 1/2-hour time frame. Parents have commented, AGAIN, please put the standings in numerical order rather than alphabetical order. Thanks!" Margaret Hoffert said, "Thank you...the questions are unpredictable and spark much discussion!" Barry Weng said, "Great questions as usual!" Denes Jakob said, "Hi Friends. As always we enjoyed the contest." Tom Wharton said, "We absolutely love this competition! Thank you for all of your efforts to make such great problems!" Ginger Moorey said, "I actually have a question for you. I have many students whose first language is NOT English. Can I define words for them (such as inscribed), give them a word list (English to Mandarin), or let them use an English to Mandarin dictionary? We really want to test the Math and they are being hindered by the Language." Excellent question, Ginger—allowing the use of an English to Mandarin dictionary would be fine.

■ Question 2-2: Comments and Appeals (Denied) Several advisers appealed on behalf of students whose answer was 25, on the theory that both the rectangle and the inscribed rhombus could effectively be the same square of side-length 5. This issue was raised by Steve Aronson, Gerry Bilodeau, Alyce Burnell, Dave Gara, Kee Ip, Denes Jakob, Mike McKay, Jon Mormino, Kevan O'Donnell, Paul Fisher, Jeff Flowers, Marissa Rakes, Brian Shay, Denise Shea, Carol Sikes, Denis J. Smith, Barry Weng, and Tom Wharton. Their reasoning is based on the warning that "the diagram is NOT drawn to scale" and requires that the triangles depicted in the diagram be degenerate triangles with one leg of length 0. The appeals are denied, as the proposed interpretation trivializes the question and contradicts the diagram. The warning about scale refers only to the relative lengths of line segments and measures of angles. Certain basic principles, for example that lines that appear straight are in fact straight, or that points that look distinct are in fact distinct, would not be affected by the scale issue. On another note, Jeff Marsh and Tom Wharton each pointed out that the "l" in "rectangle" was missing in the last word of Question 2-2. Indeed it was, Jeff and Tom—good eyes!

■ Question 2-5: Alternate Solution Margaret Hoffman reports that one of her students started by plugging in 0 and 1 for x, revealing that $f(0) = f(1) = -1/2$. Testing other numbers revealed that the value of the function is always $-1/2$.

Question 2-6: Comments and Alternate Solutions The law of cosines approach referenced in the official solution is as follows: In the diagram, at vertex O, there are 2 pairs of congruent angles. Call each of the two smaller angles x and each of the two larger angles y. Since their sum is 180 degrees, $x + y = 90$ degrees. Therefore, in right triangle BKO, angle KBO is y, so $\cos y = 1/r$.
In triangle BOC, use the law of cosines to get $12^2 = 2r^2 - 2r^2\cos y$. Substituting $1/r$ for $\cos y$ and simplifying, $72 = (r)(r - 1)$, so $r = 9$. Jon Graetz said, "If you rearrange the chords (sides of the hexagon) so that a side of length 2 is between two of the 12's, an isosceles trapezoid is inscribed in a semicircle, with bases 2r and 2, with legs of length 12. In this figure, it is easier to see that the apothem to the short chord is perpendicular to the diameter. Then the posted solution makes more sense in this picture." Wes Loewer and David Pomerance each suggested drawing line segments (apothems) from the center of the circle to the midpoints of the given sides and radii from the center to the vertices, creating right triangles of two different sizes. Wes said, "From that point, let the smaller triangles have an angle A such that $\sin(A)=1/r$, and let the larger triangles have an angle B such that $\sin(B)=6/r$. Since $2A+4B=180$ degrees, or $A+2B=90$ degrees, then $\sin^{-1}(1/r)+2*\sin^{-1}(6/r)=90$ degrees. This could be easily solved numerically on a calculator to get r=9.000, so A=81π." Wes and David each went on to show how to solve the equations. An example is as follows:

$\sin^{-1}(1/r)+2\sin^{-1}(6/r) = 90$
$2\sin^{-1}(6/r) = 90 - \sin^{-1}(1/r)$
$2\sin^{-1}(6/r) = \cos^{-1}(1/r)$
$\cos(2\sin^{-1}(6/r)) = \cos(\cos^{-1}(1/r))$
$1 - 2\sin^2(\sin^{-1}(6/r)) = 1/r$
$1 - 2(6/r)^2 = 1/r$
$1- 72/r^2 = 1/r$
$r^2 - 72 = r$
$r^2- r - 72 = 0$
$(r - 9)(r + 8) = 0$
$r = -8$ (extraneous) or 9, so $r = 9$

Statistics / Contest #2

Prob #, % Correct (*all reported scores)*

2-1	94%	2-4	31%
2-2	35%	2-5	23%
2-3	61%	2-6	6%